Probability

3 and 4

Preliminary - HSC

Name: _____

HSC CAPACITY MATRIX – GENERAL MATHEMATICS

TOPIC: Probability 3 & 4 **2 weeks**

CONTENT	CAPACITY BREAKDOWN!	DONE IT!!!!	GOT IT!!!!!	ON MY WAY!	WORKING ON IT!	HELP!!!!
1. Constructing tree diagrams	Ex 6A Q1, 3, 5, 6, 8-19					
2. Counting techniques and ordered selections	Ex 6B Ex 6C					
3. Probability trees to solve problems involving two-stage events	Ex 6D odd questions					
4. Expected number of outcomes	Medical prob. task Ex 7A S/S task - simulations					
5. Calculating financial expectation	Ex 7B					
6. Two-way tables	Ex 7C odd questions					

Your say!

What was the most important thing you learned?_____

What was something new you learnt? _____

What part(s) of this topic will you need to work on? _____

Tree diagrams

Remember:

- **In any probability experiment that has more than one stage, a tree diagram can be used to calculate the sample space;**
- **The tree diagram branches once for each stage and the number of branches at each stage is equal to the number of outcomes;**
- **The SAMPLE SPACE is found by following the path to the end of each branch;**
- **Once the sample space has been found, the probability of each outcome is calculated using the probability formula:**

$$P(event) = \frac{number\ of\ favourable\ outcomes}{total\ number\ of\ outcomes}$$

eg A coin is tossed three times. Draw a tree diagram to represent all outcomes and list the sample space.

How many stages? That's how many headings you will need.
Under the first stage, list all possible outcomes, then branch off each one.
The sample space must be explicitly presented.

eg A two digit number is formed using the digits 1, 2, 3 and 4 without repetition.

(i) Draw a tree diagram to represent all outcomes;

(ii) Calculate the probability that:

a. The number will be even;

b. The number will end in a 2;

c. The number is divisible by 3

Counting Techniques & ordered selections

A group of n different items can be arranged in n! ways.

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 1$$

eg 6! = _____

eg Nine people are standing in a line, In how many ways can the nine people be arranged?

When an ordered selection is made, the number of selections can be calculated by multiplying the number of first choices that can be made by the number of second choices that can be made and so on.
An example of an ordered selection is _____

eg In a netball team of seven players, a captain and vice captain are to be chosen. In how many ways can this be done?

HINT: Is order important? _____

To calculate the number of unordered selections that can be made, we divide the number of ordered selections by the number of arrangements of those selected.

An example of an unordered selection is

_____

eg In Lotto, a player must select six numbers out of 44. In how many ways can the six numbers be chosen?

HINT: Is order important? _____

Probability & counting techniques

eg The letters Y, C, A, T, R and E are written on separate cards. The cards are shuffled and then laid out face up in a line. Calculate the probability that the cards form the word TRACEY.

HINT: Is order important?

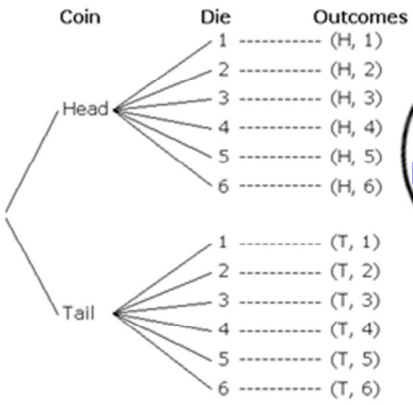
eg From Belinda, Luke, Phoebe, Elliott and Caitlin, a representative and a reserve are selected. Calculate the probability that Elliott and Luke occupy the two positions.

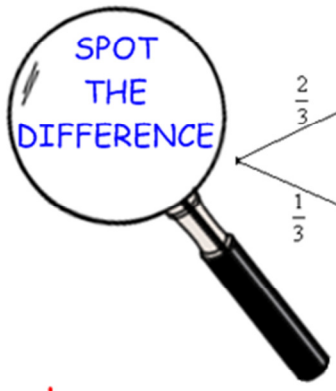
HINT: Is order important?

eg There are 45 balls in a Lotto game. Calculate the probability of winning? (You need 6 numbers in a selection)

Probability Trees

Tree Diagrams

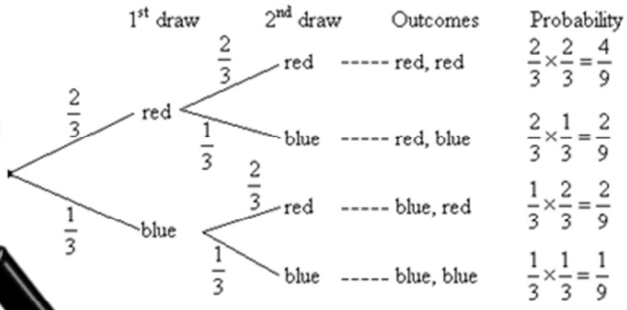




The probability of each outcome is equally likely

The probability of each element in the sample space is calculated by putting 1 over the total number of elements in the sample space.

Probability Trees



The probability of each outcome is not equally likely

The probability of each element in the sample space is calculated by MULTIPLYING along the branches.

- **IMPORTANT POINTS:**
- **READ the question CAREFULLY to note if the probabilities change during the experiment.**
- **CONSIDER what outcomes you need to include – sometimes you may need only to consider if one event occurs or not.**
- **To calculate a probability, MULTIPLY along the branches that give the required outcome.**
- **If an outcome can be obtained in two or more ways, ADD the probability of each.**
- **For questions that state "at least one", use the complementary event method: (1- P(event not occurring))**

eg In a bag there are six green marbles and eight purple marbles. A marble is drawn, colour noted and then it is replaced in the bag. A second marble is then drawn.

a) Draw a probability tree to represent the sample space;

b) Calculate the probability that:

(i) both marbles are green;

(ii) the two marbles are different colours.

eg In a barrel there are 20 white counters and thirty red counters. A counter is drawn, its colour noted and it is not replaced. A second counter is then drawn.

Find the probability that:

a) A white counter is drawn first, followed by a red counter.

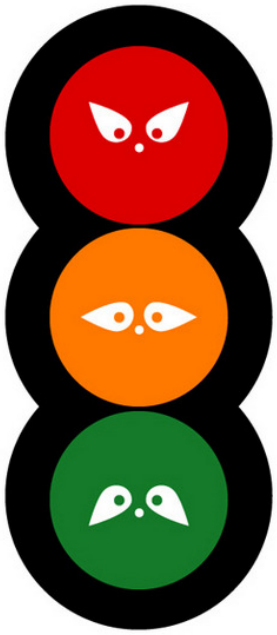
b) Both counters are the same colour.

eg Mary travels a street that has three traffic lights. The probability of catching a green light is 0.42. Calculate the probability of:

a) Catching all three green lights;

b) Catching one red light;

c) Catching at least one red light.



Expected Outcomes

The EXPECTED OUTCOME is the number of times that we expect a particular outcome to occur in a certain number of trials.
This is calculated by multiplying the probability of that event by the number of trials.
The number of times we expect an event to occur does NOT mean the event will occur that number of times. Rather, this is the average number of times we would expect this event to occur.

eg A die is rolled 150 times. How many "fives" would you expect to occur in 150 rolls of the die?

eg Ketryn draws a card from a standard deck, notes the suit and replaces the card in the deck. If Ketryn repeats this process 75 times, how many hearts can Ketryn expect to have drawn?

eg A psychologist is conducting a study on the upbringing of boys. For the study, the psychologist selects 100 couples with exactly three children. How many of these couples would the psychologist expect to have:

a) three boys;
b) two boys;
c) one boy who is the youngest.

Financial Expectation

- **Financial expectation is the average return in a financial situation.**
- **The financial expectation is calculated by multiplying each possible financial outcome by the probability of that financial outcome and adding the results together.**
- **A financial loss is indicated by a negative financial outcome while a financial gain is a positive financial outcome.**

eg A game is played in which a fair coin is tossed. If the result is a head you win $5, if the result is a tail you lose $4. Calculate the financial expectation for the game.

eg A game is played where a die is rolled. If a six is rolled, the player wins $10.00, if a five is rolled, the player wins $6.00; and if any other number is rolled, the player loses $5.00. What is the financial expectation from this game?

eg One thousand tickets are sold in a lottery. First prize is $2000, second prize is $1000 and third prize is $500. The tickets cost $10 each. If no ticket can win more than one prize, calculate the expected financial return for this lottery.

INVESTIGATION:

http://www.ildado.com/free_roulette.html

Roulette is a popular casino game played around the world. The French roulette wheel, used in Australian casinos, has 37 slots numbered from 0 to 36.

The wheel is spun and a small marble is thrown in the opposite direction. The marble comes to rest in one of the numbered slots. Apart from zero, half of the numbers are red and half are black.

They are chosen so that some are even, odd, low numbers and some are high numbers.

Bets are made by placing tokens on the appropriate places on the table. A player can bet on red numbers, black numbers, odd numbers, even numbers, pairs of numbers, groups of 6 numbers etc.

TASK 1

Complete the table below:

TYPE OF BET	WIN (for $1)	P(win)	P(loss)	Financial Expectation
Single number	$35			
Pair of adjacent numbers	$17			
3 numbers in a row	$11			
4 numbers in a row	$8			
2 rows of 3 numbers	$5			
Column of 12 numbers	$2			
1st 12 numbers	$2			
2nd 12 numbers	$2			
3rd 12 numbers	$2			
Even numbers	$1			
Odd numbers	$1			
Red numbers	$1			
Black numbers	$1			
Numbers 1-18	$1			
Numbers 19-36	$1			

a) Would it be better to bet $100 on a single number or make 100 individual $1 bets on a single number? Give reasons

TWO-WAY TABLES

- **A 2-way table is a 2-D grid that displays the outcomes of an outcome of an experiment in terms of two variables.**
- **A 2-way table is used to display test results and examine the accuracy of these results.**
- **The table displays horizontally the numbers with and without a certain condition and vertically displays information about accuracy.**
- **The table can be used to make calculations about the accuracy of the test and about the probability of those test results being accurate in an individual case.**

eg 400 newborn babies are tested for a genetic condition. The results are displayed below:

	Test results		**Total**
	Accurate	**Not accurate**	
With condition	85	9	
Without condition	304	2	
Total			

Known as a "false positive"

eg A new test was designed to assess the reading ability of students entering high school. The results were used to determine if the students' reading level was adequate to cope with high school. The students' results were then compared against existing records.

- 150 adequate readers sat for the test and 147 passed;
- 50 adequate readers sat for the test and 9 of them passed;

a) Complete the following table to represent this information

			Total
Total			

b) How many students sat the exam?
c) How many students did not pass the exam?
d) In what percentage of students who passed the test were noted as inadequate readers?

eg A batch of sniffer dogs is trained by customs to identify drugs in bags. Before they are used at airports they must pass a test. The results are given in the table below:

	Test results		**Total**
	Detected	**Not detected**	
Bags with drugs	24	1	
Bags without drugs	11	164	
Total			

a) How many bags did the sniffer dogs examine?
b) In how many bags did the dogs detect drugs?
c) Based on the above results, what is the probability that the dogs will not detect a bag carrying drugs?

PAST HSC QUESTIONS
2011

5 The letters A, B and C are used to make a three-letter company name. Each letter is used only once.

How many different company names can be made?

(A) 3

(B) 6

(C) 9

(D) 27

15 An unbiased coin is tossed 10 times.

A tail is obtained on each of the first 9 tosses.

What is the probability that a tail is obtained on the 10th toss?

(A) $\frac{1}{2^{10}}$

(B) $\frac{1}{2}$

(C) $\frac{1}{10}$

(D) $\frac{9}{10}$

(c) At another school, students who use mobile phones were surveyed. The set of data is shown in the table.

	Pre-paid	*Plan*	TOTAL
Female students	172	147	319
Male students	158	103	261
TOTAL	330	250	

(i) How many students were surveyed at this school? **1**

(ii) Of the female students surveyed, one is chosen at random. What is the probability that she uses pre-paid? **1**

(iii) Ten new male students are surveyed and all ten are on a plan. The set of data is updated to include this information. What percentage of the male students surveyed are now on a plan? Give your answer to the nearest per cent. **1**